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KINETIC CURRENT INSTABILITY OF ALFVEN WAVES IN A PLASMA WITH IONS OF FINITE LARMOR RADIUS

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SUMMARY

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This paper is devoted to the investigation of the kinetic current instability of Alfvén waves occurring in a plasma with finite Larmor radius at relative motion velocity of electron and ions exceeding the Alfvén velocity. It is serving as an example of the fact, that the effects of a finite Larmor radius of ions may appear also in a uniform but nonequilibrium plasma, for lately, particular attention has been given only to investigations of stability of inhomogenous plasma , in which the build up of oscillations takes place at the expense of the Larmor drift of particles (drift instability) at finite ratio ρ/λ.

Author

* * *

1.- The investigations of latest years have pointed to the important role of accounting the Larmor radius of ions ρ by comparison with the perturbation wavelength λ for the problem of plasma stability in a magnetic field [1, 2]. However, so far, the principal attention was given the investigation of stability of an inhomogenous plasma, in which the build up of oscillations takes place at the expense of the Larmor drift of particles (drift instability) for a finite ratio ρ/λ. Nevertheless, the effects of the finite Larmor radius of ions may also appear in a uniform, but nonequilibrium plasma (in particular at presence of particle fluxes). The kinetic current instability of Alfvén waves, occurring in a plasma with finite Larmor radius at relative

* KINETICHESKAYA TOKOVAYA NEUSTOYCHIVOST' AL'FVENOVSKIKH VOLN V PLAZME S KONECHNYM LARMOROVSKIM RADIUSOV IONOV.

motion velocities of electrons and ions (u_0), exceeding the Alfvén velocity $c_A = H_0 / (4\pi n_0 m_i)^{1/2}$, H_0 being the intensity of the magnetic field, n_0 the concentration of particles in the plasma and m_i – the ion mass), considered below, may serve as an example of that.

2. – Assume that an electric current with density $j_0 = en_0u_0$ ($-e$ being the charge of the electron) flows through a uniform low-pressure plasma ($\beta = 8\pi p / H_0^2 \ll 1$) along the magnetic field H_0. We shall estimate, that the condition $(\omega_{0i}a / c)(u_0 / c_A) < 1$ where $\omega_{0i} = (4\pi e^2 n_0 / m_i)^{1/2}$ is the ion Langmuir frequency, a is the plasma pinch, c is the speed of light, is fulfilled. This means, that the current's proper magnetic field is small by comparison with the external one. At $u_0 \sim c_A$ this condition may be written in the form $\sqrt{\beta} < \rho_i / a$ (ρ_i being the Larmor radius of ions). On the other hand, assume that $\sqrt{\beta} \gg (m_e / m_i)^{1/2}$ (m_e being the mass of the electron), that is, $c_A \ll v_e$, ($v_e = (2T_e / m_e)^{1/2}$ is the mean thermal velocity and T_e is the temperature of electrons). Evidently, both conditions may be consistent only for sufficiently great Larmor radii of ions.

Let us consider the oscillations of a plasma with frequencies $\omega \ll \omega_{Hi}$ ($\omega_{Hi} = eH_0 / m_i c$ is the ionic cyclotron frequency), propagating nearly perpendicularly to the magnetic field H_0, so that $k_\perp \gg k_\parallel$, where $k_\perp$ is the transverse and $k_\parallel$ the longitudinal component of the vector k (relative to H_0). At the same time note, that for a cylindrically-symmetric plasma pinch, we shall have, according to [2] $k_\parallel = mH_\varphi / rH_0 + k_z H_z / H_0$, where H_φ is the current's magnetic field, H_z is the external longitudinal field, $H_0 = \sqrt{H_\varphi^2 + H_z^2}$, and $m = 0, 1, 2, 3, \ldots$ However, at $j_0 = \text{const}$ ($H_\varphi \sim r$) and $H_\varphi^2 \ll H_z^2$ one may estimate $k_\parallel = \text{const}$. In this case the oscillations' dispersion equation has the form (cf. c [2])

$$\varepsilon_\parallel \left(1 - \frac{\omega^2}{k_\parallel^2 c^2}\varepsilon_\perp\right) + \frac{k_\perp^2}{k_\parallel^2}\varepsilon_\perp = 0, \tag{1}$$

where

$$\varepsilon_\perp = \frac{\omega_{0i}^2}{\omega_{Hi}^2}\frac{1 - I_0(z_i)e^{-z_i}}{z_i}; \tag{2}$$

$$\varepsilon_\parallel = \frac{2\omega_{0e}^2}{k_\parallel^2 v_e^2}[1 - x_e Y(x_e)] + \frac{2\omega_{0i}^2}{k_\parallel^2 v_i^2}[1 - x_i Y(x_i)]\, I_0(z_i)\, e^{-z_i}. \tag{3}$$

Here $\omega_{0e} = (4\pi e^2 n_0 / m_e)^{1/2}$ is the electron Langmuir frequency; $x_e = (\omega - k_\parallel u_0) / k_\parallel v_e$; $x_i = \omega / k_\parallel v_i$; $z_i = k_\perp^2 \rho_i^2 = k_\perp^2 T_i / m_i \omega_{ni}^2$; v_i is the mean thermal velocity;

T_1 is the temperature of ions; I_0 is a modified zero-index Bessel function; $Y(x) = i\sqrt{\pi}W(x)$; $W(x)$ is the "Kramp" function [2]*.

For oscillation in an isothermic plasma ($T_e \sim T_i$) with a phase longitudinal velocity lying in the interval $v_i \ll \omega / k_\parallel \ll v_e$, and at condition, that the velocity of electron flux be $u_0 \ll v_e$, the dispersion equation (1) takes the form:

$$\left(1 + i\sqrt{\pi}\,\frac{\omega - k_\parallel u_0}{k_\parallel v_e}\right)\left[1 - \frac{\omega^2}{k_\parallel{}^2 c_A{}^2}\,\frac{1 - I_0(z_i)\,e^{-z_i}}{z_i}\right] + \frac{T_e}{T_i}\left[1 - I_0(z_i)\,e^{-z_i}\right] = 0. \qquad (4)$$

It follows from (4), that

$$\mathrm{Re}\,\omega = \omega_A(\mathbf{k}) = k_\parallel c_A \left[\frac{z_i}{1 - I_0(z_i)\,e^{-z_i}} + \frac{T_e}{T_i} z_i\right]^{1/2}; \qquad (5)$$

$$\mathrm{Im}\,\omega = \gamma_A(\mathbf{k}) = -\frac{\sqrt{\pi}}{2}\,\frac{T_e}{T_i}\,z_i\,\frac{k_\parallel{}^2 c_A{}^2}{\omega_A(\mathbf{k})}\,\frac{\omega_A(\mathbf{k}) - k_\parallel u_0}{k_\parallel v_e}, \qquad (6)$$

with, at the same time, $\gamma_A \ll \omega_A$. At $z_i = k_\perp{}^2 \rho_i{}^2 \to 0$ formula (5) passes into the well known expression for the Alfvén wave frequency $\omega_A = k_\parallel c_A$, and the decrement (increment) $\gamma_A \to 0$. At finite Larmor radii of ions ($z_i \sim 1$) and at sufficiently great velocities of the relative motion of electrons and ions ($u_0 > c_A$), the Alfvén oscillations become unsteady ($\gamma_A > 0$).

3.- We shall briefly discuss the physical nature of that instability. At usual hydrodynamic consideration of Alfvén oscillations, when $\rho / \lambda \to 0$, it is found that their electric field is directed across the constant magnetic field ($E \perp H_0$). Because of that, the interaction of the wave with particles is ($E_\perp j_\parallel = 0$), so that in this approximation the Alfvén oscillations are nondamping, contrary to other types of oscillations (for example those of ionic or magnetic sound [3]).

The situation changes substantially, however, if we take into account the finiteness of the Larmor radius of ions. There appears then in Alfvén waves a longitudinal component of the electric field which is not zero $E_\parallel = -(k_\perp / k_\parallel)(\varepsilon_\perp / \varepsilon_\parallel) E_\perp \approx -(k_\parallel / k_\perp)(T_e / T_i) z_i E_\perp \neq 0$. As a result, the Alfvén waves, interacting with resonance particles (electrons), may either damp (Landau damping in equilibrium plasma or at $u_0 < C_A$) or build up (current instability $\gamma_A > 0$ at $u_0 > C_A$). It is interesting to note, that in the given case build up of magnetohydrodynamic oscillations (macroscopic) takes place at the expense of kinetic-type instability, which may be manifest

* [in transliteration].

in the retention of plasma by the magnetic field.

4.- Note that at $z_i = k_\perp^2 \rho_i^2 \gtrsim 1$ and for sufficient great $k_\parallel$, the frequency of Alfvén oscillations may become of the order of ion cyclotron frequency ($\omega_A \sim \omega_{Hi}$). In this case, we should substitute in the equation (1) the following expression for $\varepsilon_\perp$, in place of (2):

$$\varepsilon_\perp = \frac{c^2}{c_A^2}\frac{1}{z_i}\left\{1 - I_0(z_i)\,e^{-z_i} - \sum_{n=-\infty}^{\infty}\int \frac{\omega I_n(z_i)\,e^{-z_i} f_\parallel^i(v_\parallel)\,dv_\parallel}{\omega - n\omega_{ni} - k_\parallel v_\parallel}\right\}, \qquad (7)$$

where $f_\parallel^i$ is the "longitudinal" ionic distribution function (the distribution of ions is assumed isotropic and Maxwellian).

Then, at the condition that $k_\parallel v_i \ll |\omega - n\omega_{ni}|$, the dispersion equation (2), near the n-th cyclotron harmonic takes the form:

$$1 - I_0(z_i)\,e^{-z_i} + \frac{T_i}{T_e}\left[1 + i\sqrt{\pi}\,\frac{\omega - k_\parallel u_0}{k_\parallel v_e}\right] \Big/ \left[1 - \frac{\omega^2}{(T_e/T_i)\,z_i k_\parallel^2 c_A^2} \times \right.$$
$$\left. \times \left(1 + i\sqrt{\pi}\,\frac{\omega - k_\parallel u_0}{k_\parallel v_e}\right)\right] = \frac{\omega I_n(z_i)\,e^{-z_i}}{\omega - n\omega_{Hi}}; \qquad (8)$$

where I_n is the modified Bessel function of the n-th index ($n = 1, 2, 3 \ldots$).

If we assume that $\omega^2 \sim (n\omega_{Hi})^2 \ll (T_e/T_i) z_i k_\parallel^2 c_A^2$, the equation (8) passes into the dispersion equation of the work [4], where the current instability of potential cyclotron oscillations was considered. At $k_\parallel u_0 \sim n\omega_{Hi}$, $T_e \sim T_i$ and $z_i \sim 1$: hence it follows, that $u_0^2 \ll c_A^2$. But, on the other hand, from the condition of smallness of ion damping it follows that $u_0^2/c_A^2 > 10^2\beta$ [4]. Therefore, the assumption of potentiality of oscillations, made in [4], is valid only in the case of sufficiently rarefied plasma, when $\beta \ll 10^{-2}$.

The accretion increment of nonpotential ($\beta \gg m_e/m_i$) cyclotron oscillations ($\omega \approx n\omega_{Hi}$), is, according to (8):

$$\gamma_n \cong \sqrt{\pi}\, n\omega_{Hi}\left(\frac{u_0}{v_e} - \frac{n\omega_{Hi}}{k_\parallel v_e}\right)\frac{T_e}{T_i} I_n(z_i)\,e^{-z_i} \times$$
$$\times \left\{1 + \frac{T_e}{T_i}\left[1 - I_0(z_i)\,e^{-z_i}\right]\left(1 - \frac{n^2\omega_{Hi}^2}{(T_e/T_i) z_i k_\parallel^2 c_A^2}\right)\right\}^{-2}. \qquad (9)$$

It approaches the maximum, formally infinite, precisely at the condition that the ion cyclotron frequency ω_{Hi} (or one of its harmonics $n\omega_{Hi}$) is equal to the frequency of Alfvén oscillations $\omega_A(k)$. The joint consideration of cyclotron and Alfvén branches of oscillations near their

point of intersection provides the final maximum value of the increment for cyclotron oscillations :

$$\gamma_{n\max}^{(1,2)} \cong \frac{\sqrt{\pi}}{2} n\omega_{Hi}\left(\frac{u_0}{v_e} - \frac{n\omega_{Hi}}{k_\parallel v_e}\right)\left\{\frac{1}{2}\frac{1 - I_0(z_i)e^{-z_i}}{2 - I_0(z_i)e^{-z_i}} + \right.$$
$$\left. + \left[\frac{1}{2}\frac{I_n(z_i)e^{-z_i}}{1 - I_0(z_i)e^{-z_i}}\right]^{1/3}\left[2 - I_0(z_i)e^{-z_i}\right]^{-1/3}\right\} \qquad (10)$$

(for the sake of simplicity we consider $T_e = T_i$). Hence, it follows, in particular, that as a result of interaction of Alfvén oscillations with cyclotron harmonics, the increment of current instability of oscillations with $\omega \approx n\omega_{Hi}$ may be substantially greater than the increment of potential cyclotron oscillations [4].

In conclusion, we extend our thanks to B. B. Kadomtsev for discussing the results and providing useful counsel.

*** THE END ***

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